1. Find all positive integers $n$ such $20n + 2$ can divide $2003n + 2002$.

2. There are $3n, n \in \mathbb{Z}^+$ girl students who took part in a summer camp. There were three girl students to be on duty every day. When the summer camp ended, it was found that any two of the $3n$ students had just one time to be on duty on the same day.
   (1) When $n = 3$, is there any arrangement satisfying the requirement above. Prove your conclusion.
   (2) Prove that $n$ is an odd number.

3. Find all positive integers $k$ such that for any positive numbers $a, b$ and $c$ satisfying the inequality

$$k(ab + bc + ca) > 5(a^2 + b^2 + c^2),$$

there must exist a triangle with $a, b$ and $c$ as the length of its three sides respectively.

4. Circles $O_1$ and $O_2$ intersect at two points $B$ and $C$, and $BC$ is the diameter of circle $O_1$. Construct a tangent line of circle $O_1$ at $C$ and intersecting circle $O_2$ at another point $A$. We join $AB$ to intersect circle $O_1$ at point $E$, then join $CE$ and extend it to intersect circle $O_2$ at point $F$. Assume $H$ is an arbitrary point on line segment $AF$. We join $HE$ and extend it to intersect circle $O_1$ at point $G$, and then join $BG$ and extend it to intersect the extend line of $AC$ at point $D$. Prove:

$$\frac{AH}{HF} = \frac{AC}{CD}.$$
Day 2

1. There are \( n \geq 2 \) permutations \( P_1, P_2, \ldots, P_n \) each being an arbitrary permutation of \( \{1, \ldots, n\} \). Prove that

\[
\sum_{i=1}^{n-1} \frac{1}{P_i + P_{i+1}} > \frac{n-1}{n+2}.
\]

2. Find all pairs of positive integers \((x, y)\) such that

\[
x^y = y^{x-y}.
\]

Albania

3. An acute triangle \( ABC \) has three heights \( AD, BE \) and \( CF \) respectively. Prove that the perimeter of triangle \( DEF \) is not over half of the perimeter of triangle \( ABC \).

4. Assume that \( A_1, A_2, \ldots, A_8 \) are eight points taken arbitrarily on a plane. For a directed line \( l \) taken arbitrarily on the plane, assume that projections of \( A_1, A_2, \ldots, A_8 \) on the line are \( P_1, P_2, \ldots, P_8 \) respectively. If the eight projections are pairwise disjoint, they can be arranged as \( P_{i_1}, P_{i_2}, \ldots, P_{i_8} \) according to the direction of line \( l \). Thus we get one permutation for \( 1, 2, \ldots, 8 \), namely, \( i_1, i_2, \ldots, i_8 \). In the figure, this permutation is 2, 1, 8, 3, 7, 4, 6, 5. Assume that after these eight points are projected to every directed line on the plane, we get the number of different permutations as \( N_8 = N(A_1, A_2, \ldots, A_8) \). Find the maximal value of \( N_8 \).
Day 1

1. Let \(ABC\) be a triangle. Points \(D\) and \(E\) are on sides \(AB\) and \(AC\), respectively, and point \(F\) is on line segment \(DE\). Let \(\frac{AD}{AB} = x\), \(\frac{AE}{AC} = y\), \(\frac{DF}{DE} = z\). Prove that
   \[(1) \ S_{\triangle BDF} = (1-x)y S_{\triangle ABC} \text{ and } S_{\triangle CEF} = x(1-y)(1-z) S_{\triangle ABC};
   \]
   \[(2) \ \sqrt[3]{S_{\triangle BDF}} + \sqrt[3]{S_{\triangle CEF}} \leq \sqrt[3]{S_{\triangle ABC}}. \]

2. There are 47 students in a classroom with seats arranged in 6 rows \(\times\) 8 columns, and the seat in the \(i\)-th row and \(j\)-th column is denoted by \((i,j)\). Now, an adjustment is made for students seats in the new school term. For a student with the original seat \((i,j)\), if his/her new seat is \((m,n)\), we say that the student is moved by \([a,b] = [i-m, j-n]\) and define the position value of the student as \(a+b\). Let \(S\) denote the sum of the position values of all the students. Determine the difference between the greatest and smallest possible values of \(S\).

3. As shown in the figure, quadrilateral \(ABCD\) is inscribed in a circle with \(AC\) as its diameter, \(BD \perp AC\), and \(E\) the intersection of \(AC\) and \(BD\). Extend line segment \(DA\) and \(BA\) through \(A\) to \(F\) and \(G\) respectively, such that \(DG \parallel BF\). Extend \(GF\) to \(H\) such that \(CH \perp GH\). Prove that points \(B, E, F\) and \(H\) lie on one circle.

4. (1) Prove that there exist five nonnegative real numbers \(a, b, c, d\) and \(e\) with their sum equal to 1 such that for any arrangement of these numbers around a circle, there are always two neighboring numbers with their product not less than \(\frac{1}{9}\).
   
   (2) Prove that for any five nonnegative real numbers with their sum equal to 1, it is always possible to arrange them around a circle such that there are two neighboring numbers with their product not greater than \(\frac{1}{9}\).
Day 2

1. Let \( \{a_n\}_{n=1}^{\infty} \) be a sequence of real numbers such that \( a_1 = 2 \), and

\[
a_{n+1} = a_n^2 - a_n + 1, \quad \forall n \in \mathbb{N}.
\]

Prove that

\[
1 - \frac{1}{2003^{2003}} < \sum_{i=1}^{2003} \frac{1}{a_i} < 1.
\]

2. Let \( n \geq 2 \) be an integer. Find the largest real number \( \lambda \) such that the inequality

\[
a_n^2 \geq \lambda \sum_{i=1}^{n-1} a_i + 2 \cdot a_n.
\]

holds for any positive integers \( a_1, a_2, \ldots, a_n \) satisfying \( a_1 < a_2 < \ldots < a_n \).

3. Let the sides of a scalene triangle \( \triangle ABC \) be \( AB = c, \ BC = a, \ CA = b \), and \( D, E, F \) be points on \( BC, CA, AB \) such that \( AD, BE, CF \) are angle bisectors of the triangle, respectively. Assume that \( DE = DF \). Prove that

(1) \( \frac{a}{b+c} = \frac{b}{c+a} + \frac{c}{a+b} \)

(2) \( \angle BAC > 90^\circ \).

4. Let \( n \) be a positive integer, and \( S_n \), be the set of all positive integer divisors of \( n \) (including 1 and itself). Prove that at most half of the elements in \( S_n \) have their last digits equal to 3.
Day 1

1. We say a positive integer \( n \) is *good* if there exists a permutation \( a_1, a_2, \ldots, a_n \) of \( 1, 2, \ldots, n \) such that \( k + a_k \) is perfect square for all \( 1 \leq k \leq n \). Determine all the good numbers in the set \( \{11, 13, 15, 17, 19\} \).

2. Let \( a, b, c \) be positive reals. Find the smallest value of

\[
\frac{a + 3c}{a + 2b + c} + \frac{4b}{a + b + 2c} - \frac{8c}{a + b + 3c}.
\]

3. Let \( ABC \) be an obtuse inscribed in a circle of radius 1. Prove that \( \triangle ABC \) can be covered by an isosceles right-angled triangle with hypotenuse of length \( \sqrt{2} + 1 \).

4. A deck of 32 cards has 2 different jokers each of which is numbered 0. There are 10 red cards numbered 1 through 10 and similarly for blue and green cards. One chooses a number of cards from the deck. If a card in hand is numbered \( k \), then the value of the card is \( 2^k \), and the value of the hand is sum of the values of the cards in hand. Determine the number of hands having the value 2004.
Day 2

1. Let $u, v, w$ be positive real numbers such that $u\sqrt{vw} + v\sqrt{wu} + w\sqrt{uv} \geq 1$. Find the smallest value of $u + v + w$.

2. Given an acute triangle $ABC$ with $O$ as its circumcenter. Line $AO$ intersects $BC$ at $D$. Points $E, F$ are on $AB, AC$ respectively such that $A, E, D, F$ are concyclic. Prove that the length of the projection of line segment $EF$ on side $BC$ does not depend on the positions of $E$ and $F$.

3. Let $p$ and $q$ be two coprime positive integers, and $n$ be a non-negative integer. Determine the number of integers that can be written in the form $ip + jq$, where $i$ and $j$ are non-negative integers with $i + j \leq n$.

4. When the unit squares at the four corners are removed from a three by three squares, the resulting shape is called a cross. What is the maximum number of non-overlapping crosses placed within the boundary of a $10 \times 11$ chessboard? (Each cross covers exactly five unit squares on the board.)
Day 1

1. As shown in the following figure, point $P$ lies on the circumcircle of triangle $ABC$. Lines $AB$ and $CP$ meet at $E$, and lines $AC$ and $BP$ meet at $F$. The perpendicular bisector of line segment $AB$ meets line segment $AC$ at $K$, and the perpendicular bisector of line segment $AC$ meets line segment $AB$ at $J$. Prove that

\[
\frac{(CE)^2}{BF} = \frac{AJ \cdot JE}{AK \cdot KF}.
\]

2. Find all ordered triples $(x, y, z)$ of real numbers such that

\[
5 \left( x + \frac{1}{x} \right) = 12 \left( y + \frac{1}{y} \right) = 13 \left( z + \frac{1}{z} \right),
\]

and

\[
xy + yz + zy = 1.
\]

3. Determine if there exists a convex polyhedron such that
   (1) it has 12 edges, 6 faces and 8 vertices;
   (2) it has 4 faces with each pair of them sharing a common edge of the polyhedron.

4. Determine all positive real numbers $a$ such that there exists a positive integer $n$ and sets $A_1, A_2, \ldots, A_n$ satisfying the following conditions:
   (1) every set $A_i$ has infinitely many elements;
   (2) every pair of distinct sets $A_i$ and $A_j$ do not share any common element
   (3) the union of sets $A_1, A_2, \ldots, A_n$ is the set of all integers;
   (4) for every set $A_i$, the positive difference of any pair of elements in $A_i$ is at least $a^i$. 
Day 2

1. Let $x$ and $y$ be positive real numbers with $x^3 + y^3 = x - y$. Prove that
   
   $x^2 + 4y^2 < 1$.

2. An integer $n$ is called good if there are $n \geq 3$ lattice points $P_1, P_2, \ldots, P_n$ in the coordinate plane satisfying the following conditions: If line segment $P_iP_j$ has a rational length, then there is $P_k$ such that both line segments $P_iP_k$ and $P_jP_k$ have irrational lengths; and if line segment $P_iP_j$ has an irrational length, then there is $P_k$ such that both line segments $P_iP_k$ and $P_jP_k$ have rational lengths.

   (1) Determine the minimum good number.

   (2) Determine if 2005 is a good number. (A point in the coordinate plane is a lattice point if both of its coordinate are integers.)

3. Let $m$ and $n$ be positive integers with $m > n \geq 2$. Set $S = \{1, 2, \ldots, m\}$, and $T = \{a_1, a_2, \ldots, a_n\}$ is a subset of $S$ such that every number in $S$ is not divisible by any two distinct numbers in $T$. Prove that

   \[
   \sum_{i=1}^{n} \frac{1}{a_i} < \frac{m + n}{m}.
   \]

4. Given an $a \times b$ rectangle with $a > b > 0$, determine the minimum length of a square that covers the rectangle. (A square covers the rectangle if each point in the rectangle lies inside the square.)
Day 1

1. Let \( a > 0 \), the function \( f : (0, +\infty) \to \mathbb{R} \) satisfies \( f(a) = 1 \), if for any positive reals \( x \) and \( y \), there is
   \[
   f(x)f(y) + f\left(\frac{a}{x}\right)f\left(\frac{a}{y}\right) = 2f(xy)
   \]
   then prove that \( f(x) \) is a constant.

2. Let \( O \) be the intersection of the diagonals of convex quadrilateral \( ABCD \). The circumcircles of \( \triangle OAD \) and \( \triangle OBC \) meet at \( O \) and \( M \). Line \( OM \) meets the circumcircles of \( \triangle OAB \) and \( \triangle OCD \) at \( T \) and \( S \) respectively.
   Prove that \( M \) is the midpoint of \( ST \).

3. Show that for any \( i = 1, 2, 3 \), there exist infinity many positive integer \( n \), such that among \( n, n + 2 \) and \( n + 28 \), there are exactly \( i \) terms that can be expressed as the sum of the cubes of three positive integers.

4. 8 people participate in a party.
   (1) Among any 5 people there are 3 who pairwise know each other. Prove that there are 4 people who pairwise know each other.
   (2) If among any 6 people there are 3 who pairwise know each other, then can we find 4 people who pairwise know each other?
Day 2

1. The set $S = \{(a, b) \mid 1 \leq a, b \leq 5, a, b \in \mathbb{Z}\}$ be a set of points in the plane with integral coordinates. $T$ is another set of points with integral coordinates in the plane. If for any point $P \in S$, there is always another point $Q \in T$, $P \neq Q$, such that there is no other integral points on segment $PQ$. Find the least value of the number of elements of $T$.

2. Let $M = \{1, 2, \cdots, 19\}$ and $A = \{a_1, a_2, \cdots, a_k\} \subseteq M$. Find the least $k$ so that for any $b \in M$, there exist $a_i, a_j \in A$, satisfying $b = a_i$ or $b = a_i \pm a_i$ ($a_i$ and $a_j$ do not have to be different).

3. Given that $x_i > 0, i = 1, 2, \cdots, n, k \geq 1$. Show that:

$$\sum_{i=1}^{n} \frac{1}{1 + x_i} \cdot \sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} \frac{x_i^{k+1}}{1 + x_i} \cdot \sum_{i=1}^{n} \frac{1}{x_i^k}$$

4. Let $p$ be a prime number that is greater than 3. Show that there exist some integers $a_1, a_2, \cdots, a_k$ that satisfy:

$$-\frac{p}{2} < a_1 < a_2 < \cdots < a_k < \frac{p}{2}$$

making the product:

$$\frac{p - a_1}{|a_1|} \cdot \frac{p - a_2}{|a_2|} \cdot \cdots \cdot \frac{p - a_k}{|a_k|}$$

equals to $3^m$ where $m$ is a positive integer.
As a high school student, competing in mathematics competitions, I enjoyed mathematics as a sport, taking cleverly designed mathematical puzzle problems and searching for the right “trick” that would unlock each one. As an undergraduate, I was awed by my first glimpses of the rich, deep, and fascinating theories and structures which lie at the core of modern mathematics today … … how one approaches a mathematical problem for the first time, and how the painstaking, systematic experience of trying some ideas, eliminating others, and steadily manipulating the problem can lead, ultimately, to a satisfying solution.

from Solving Mathematical Problems: A Personal Perspective, by Terence Tao

1. A positive integer $m$ is called good if there is a positive integer $n$ such that $m$ is the quotient of $n$ by the number of positive integer divisors of $n$ (including 1 and $n$ itself). Prove that $1, 2, \ldots, 17$ are good numbers and that 18 is not a good number.

2. Let $ABC$ be an acute triangle. Points $D, E,$ and $F$ lie on segments $BC, CA,$ and $AB$, respectively, and each of the three segments $AD, BE,$ and $CF$ contains the circumcenter of $ABC$. Prove that if any two of the ratios $BD/DC', CE/EA', AF/BF', AE/FA', EC'/DB, CD/DB$ are integers, then triangle $ABC$ is isosceles.

3. Let $n$ be an integer greater than 3, and let $a_1, a_2, \ldots, a_n$ be nonnegative real numbers with $a_1 + a_2 + \cdots + a_n = 2$. Determine the minimum value of $rac{a_1}{a_2^2 + 1} + \frac{a_2}{a_3^2 + 1} + \cdots + \frac{a_n}{a_1^2 + 1}$.

4. The set $S$ consists of $n > 2$ points in the plane. The set $P$ consists of $m$ lines in the plane such that every line in $P$ is an axis of symmetry for $S$. Prove that $m \leq n$, and determine when equality holds.

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But I just like mathematics because it’s fun.

Mathematics problems, or puzzles, are important to real mathematics (like solving real-life problems), just as fables, stories and anecdotes are important to young in understanding real life. Mathematical problems are “sanitized” mathematics, where an elegant solutions has already been found, the question is stripped of all superfluousness and posed in an interesting and thought-provoking way.

from Solving Mathematical Problems: A Personal Perspective, by Terence Tao

5. Point \(D\) lies inside triangle \(ABC\) such that \(\angle DAC = \angle DCA = 30^\circ\) and \(\angle DBA = 60^\circ\). Point \(E\) is the midpoint of segment \(BC\). Point \(F\) lies on segment \(AC\) with \(AF = 2FC\). Prove that \(DE \perp EF\).

6. For nonnegative real numbers \(a, b, c\) with \(a + b + c = 1\), prove that

\[
\sqrt{a + \frac{(b - c)^2}{4}} + \sqrt{b} + \sqrt{c} \leq \sqrt{3}.
\]

7. Let \(a, b, c\) be integers each with absolute value less than or equal to 10. The cubic polynomial \(f(x) = x^3 + ax^2 + bx + c\) satisfies the property

\[
|f(2 + \sqrt{3})| < 0.0001.
\]

Determine if \(2 + \sqrt{3}\) is a root of \(f\).

8. In a round robin chess tournament each player plays every other player exactly once. The winner of each game gets 1 point and the loser gets 0 points. If the game is tied, each player gets 0.5 points.

Given a positive integer \(m\), a tournament is said to have property \(P(m)\) if the following holds: among every set \(S\) of \(m\) players, there is one player who won all her games against the other \(m - 1\) players in \(S\) and one player who lost all her games against the other \(m - 1\) players in \(S\).

For a given integer \(m \geq 4\), determine the minimum value of \(n\) (as a function of \(m\)) such that the following holds: in every \(n\)-player round robin chess tournament with property \(P(m)\), the final scores of the \(n\) players are all distinct.
1. (a) Determine if the set \{1, 2, \ldots, 96\} can be partitioned into 32 sets of equal size and equal sum.

(b) Determine if the set \{1, 2, \ldots, 99\} can be partitioned into 33 sets of equal size and equal sum.

2. Let \( \phi(x) = ax^3 + bx^2 + cx + d \) be a polynomial with real coefficients. Given that \( \phi(x) \) has three positive real roots and that \( \phi(0) < 0 \), prove that
\[
2b^3 + 9a^2d - 7abc \leq 0.
\]

3. Determine the least real number \( a \) greater than 1 such that for any point \( P \) in the interior of square \( ABCD \), the area ratio between two of the triangles \( PAB, PBC, PCD, PDA \) lies in the interval \( \left[ \frac{1}{a}, a \right] \).

4. Equilateral triangles \( ABQ, BCR, CDS, DAP \) are erected outside of the (convex) quadrilateral \( ABCD \). Let \( X, Y, Z, W \) be the midpoints of the segments \( PQ, QR, RS, SP \), respectively. Determine the maximum value of
\[
\frac{XZ + YW}{AC + BD}.
\]

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5. In (convex) quadrilateral $ABCD$, $AB = BC$ and $AD = DC$. Point $E$ lies on segment $AB$ and point $F$ lies on segment $AD$ such that $B, E, F, D$ lie on a circle. Point $P$ is such that triangles $DPE$ and $ADC$ are similar and the corresponding vertices are in the same orientation (clockwise or counterclockwise). Point $Q$ is such that triangles $BQF$ and $ABC$ are similar and the corresponding vertices are in the same orientation. Prove that points $A, P, Q$ are collinear.

6. Let $(x_1, x_2, \ldots)$ be a sequence of positive numbers such that $(8x_2 - 7x_1)x_1^2 = 8$ and

$$x_{k+1}x_{k-1} - x_k^2 = \frac{x_{k-1}^2 - x_k^2}{x_k^2} \text{ for } k = 2, 3, \ldots$$

Determine real number $a$ such that if $x_1 > a$, then the sequence is monotonically decreasing, and if $0 < x_1 < a$, then the sequence is not monotonic.

7. On a given $2008 \times 2008$ chessboard, each unit square is colored in a different color. Every unit square is filled with one of the letters $C, G, M, O$. The resulting board is called harmonic if every $2 \times 2$ subsquare contains all four different letters. How many harmonic boards are there?

8. For positive integers $n$, $f_n = \left\lfloor 2^n \cdot \sqrt{2008} \right\rfloor + \left\lfloor 2^n \cdot \sqrt{2009} \right\rfloor$. Prove that there are infinitely many odd numbers and infinitely many even numbers in the sequence $f_1, f_2, \ldots$. 
Day 1

1. Show that there are only finitely many triples \((x, y, z)\) of positive integers satisfying the equation \(abc = 2009(a + b + c)\).

2. Right triangle \(ABC\), with \(\angle A = 90^\circ\), is inscribed in circle \(\Gamma\). Point \(E\) lies on the interior of arc \(BC\) (not containing \(A\)) with \(EA > EC\). Point \(F\) lies on ray \(EC\) with \(\angle EAC = \angle CAF\). Segment \(BF\) meets \(\Gamma\) again at \(D\) (other than \(B\)). Let \(O\) denote the circumcenter of triangle \(DEF\). Prove that \(A, C, O\) are collinear.

3. Let \(n\) be a given positive integer. In the coordinate set, consider the set of points \(\{P_1, P_2, ..., P_{4n+1}\} = \{(x, y)| x, y \in \mathbb{Z}, xy = 0, |x| \leq n, |y| \leq n\}\). Determine the minimum of \((P_1P_2)^2 + (P_2P_3)^2 + ... + (P_{4n}P_{4n+1})^2 + (P_{4n+1}P_1)^2\).

4. Let \(n\) be an integer greater than 3. Points \(V_1, V_2, ..., V_n\), with no three collinear, lie on a plane. Some of the segments \(V_iV_j\), with \(1 \leq i < j \leq n\), are constructed. Points \(V_i\) and \(V_j\) are neighbors if \(V_iV_j\) is constructed. Initially, chess pieces \(C_1, C_2, ..., C_n\) are placed at points \(V_1, V_2, ..., V_n\) (not necessarily in that order) with exactly one piece at each point. In a move, one can choose some of the \(n\) chess pieces, and simultaneously relocate each of the chosen piece from its current position to one of its neighboring positions such that after the move, exactly one chess piece is at each point and no two chess pieces have exchanged their positions.

A set of constructed segments is called harmonic if for any initial positions of the chess pieces, each chess piece \(C_i\) (\(1 \leq i \leq n\)) is at the point \(V_i\) after a finite number of moves. Determine the minimum number of segments in a harmonic set.
Day 2

1. Let $x, y, z$ be real numbers greater than or equal to 1. Prove that
   \[ \prod (x^2 - 2x + 2) \leq (xyz)^2 - 2xyz + 2. \]

2. Circle $\Gamma_1$, with radius $r$, is internally tangent to circle $\Gamma_2$ at $S$. Chord $AB$ of $\Gamma_2$ is tangent to $\Gamma_1$ at $C$. Let $M$ be the midpoint of arc $AB$ (not containing $S$), and let $N$ be the foot of the perpendicular from $M$ to line $AB$. Prove that $AC \cdot CB = 2r \cdot MN$.

3. On a $10 \times 10$ chessboard, some $4n$ unit squares are chosen to form a region $R$. This region $R$ can be tiled by $n$ $2 \times 2$ squares. This region $R$ can also be tiled by a combination of $n$ pieces of the following types of shapes (see below, with rotations allowed). Determine the value of $n$.

4. For a positive integer $n$, $a_n = n\sqrt{5} - \lfloor n\sqrt{5} \rfloor$. Compute the maximum value and the minimum value of $a_1, a_2, ..., a_{2009}$.
1. Let $n$ be an integer greater than two, and let $A_1, A_2, \ldots, A_{2n}$ be pairwise distinct nonempty subsets of $\{1, 2, \ldots, n\}$. Determine the maximum value of

$$\sum_{i=1}^{2n} \frac{|A_i \cap A_{i+1}|}{|A_i| \cdot |A_{i+1}|}.\]

(Here we set $A_{2n+1} = A_1$. For a set $X$, let $|X|$ denote the number of elements in $X$.)

2. In triangle $ABC$, $AB = AC$. Point $D$ is the midpoint of side $BC$. Point $E$ lies outside the triangle $ABC$ such that $CE \perp AB$ and $BE = BD$. Let $M$ be the midpoint of segment $BE$. Point $F$ lies on the minor arc $AD$ of the circumcircle of triangle $ABD$ such that $MF \perp BE$. Prove that $ED \perp FD$.

3. Prove that for every given positive integer $n$, there exists a prime $p$ and an integer $m$ such that

(a) $p \equiv 5 \pmod{6}$;
(b) $p \nmid n$;
(c) $n \equiv m^3 \pmod{p}$.

4. Let $x_1, x_2, \ldots, x_n$ (where $n \geq 2$) be real numbers with $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$. Prove that

$$\sum_{k=1}^{n} \left(1 - \frac{k}{\sum_{i=1}^{n} i x_i^2}\right)^2 \cdot \frac{x_k^2}{k} \leq \left(\frac{n-1}{n+1}\right)^2 \sum_{k=1}^{n} \frac{x_k^2}{k}.$$ Determine when does the equality hold.

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5. Let \( f(x) \) and \( g(x) \) be strictly increasing linear functions from \( \mathbb{R} \) to \( \mathbb{R} \) such that \( f(x) \) is an integer if and only if \( g(x) \) is an integer. Prove that for any real number \( x \), \( f(x) - g(x) \) is an integer.

6. In acute triangle \( ABC \), \( AB > AC \). Let \( M \) be the midpoint of side \( BC \). The exterior angle bisector of \( \angle BAC \) meets ray \( BC \) at \( P \). Points \( K \) and \( F \) lie on the line \( PA \) such that \( MF \perp BC \) and \( MK \perp PA \). Prove that \( BC^2 = 4PF \cdot AK \).

7. Let \( n \) be an integer greater than or equal to 3. For a permutation \( p = (x_1, x_2, \ldots, x_n) \) of \( (1, 2, \ldots, n) \), we say that \( x_j \) lies in between \( x_i \) and \( x_k \) if \( i < j < k \). (For example, in the permutation \( (1, 3, 2, 4) \), 3 lies in between 1 and 4, and 4 does not lie in between 1 and 2.) Set \( S = \{p_1, p_2, \ldots, p_m\} \) consists of (distinct) permutations \( p_i \) of \( (1, 2, \ldots, n) \). Suppose that among every three distinct numbers in \( \{1, 2, \ldots, n\} \), one of these numbers does not lie in between the other two numbers in every permutations \( p_i \in S \). Determine the maximum value of \( m \).

8. Determine the least odd number \( a > 5 \) satisfying the following conditions: There are positive integers \( m_1, m_2, n_1, n_2 \) such that \( a = m_1^2 + n_1^2 \), \( a^2 = m_2^2 + n_2^2 \), and \( m_1 - n_1 = m_2 - n_2 \).
Day 1

1. Find all positive integers $n$ such that the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ has exactly 2011 positive integer solutions $(x, y)$ where $x \leq y$.

2. The diagonals $AC, BD$ of the quadrilateral $ABCD$ intersect at $E$. Let $M, N$ be the midpoints of $AB, CD$ respectively. Let the perpendicular bisectors of the segments $AB, CD$ meet at $F$. Suppose that $EF$ meets $AD, BC$ at $P, Q$ respectively. If $MF \cdot CD = NF \cdot AB$ and $DQ \cdot BP = AQ \cdot CP$, prove that $PQ \perp BC$.

3. The positive reals $a, b, c, d$ satisfy $abcd = 1$. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{9}{a+b+c+d} \geq \frac{25}{4}$.

4. A tennis tournament has $n > 2$ players and any two players play one game against each other (ties are not allowed). After the game these players can be arranged in a circle, such that for any three players $A, B, C$, if $A, B$ are adjacent on the circle, then at least one of $A, B$ won against $C$. Find all possible values for $n$. 
Day 2

1. A real number $\alpha \geq 0$ is given. Find the smallest $\lambda = \lambda(\alpha) > 0$, such that for any complex numbers $z_1, z_2$ and $0 \leq x \leq 1$, if $|z_1| \leq \alpha |z_1 - z_2|$, then $|z_1 - xz_2| \leq \lambda |z_1 - z_2|$.

2. Do there exist positive integers $m, n$, such that $m^{20} + 11^n$ is a square number?

3. There are $n$ boxes $B_1, B_2, \ldots, B_n$ from left to right, and there are $n$ balls in these boxes. If there is at least 1 ball in $B_1$, we can move one to $B_2$. If there is at least 1 ball in $B_n$, we can move one to $B_{n-1}$. If there are at least 2 balls in $B_k$, $2 \leq k \leq n - 1$ we can move one to $B_{k-1}$, and one to $B_{k+1}$. Prove that, for any arrangement of the $n$ balls, we can achieve that each box has one ball in it.

4. The $A$-excircle $(O)$ of $\triangle ABC$ touches $BC$ at $M$. The points $D, E$ lie on the sides $AB, AC$ respectively such that $DE \parallel BC$. The incircle $(O_1)$ of $\triangle ADE$ touches $DE$ at $N$. If $BO_1 \cap DO = F$ and $CO_1 \cap EO = G$, prove that the midpoint of $FG$ lies on $MN$. 
